

1 Introduction

What does it take to build a professional soccer team that wins titles? The Premier League, the world's largest soccer league, has seen 51 different teams since it's inaugural season but only 7 of them have won. Why is it that so few teams have won? One of the many issues with building a team is that you're never sure how well players will play together and how they fit into your team. This process requires a collaborative effort from a dedicated team of personnel to evaluate a player and decide whether they should pursue them. Every player possesses a market value which is the estimated price a team would have to pay to purchase a player. This price is influenced by several factors, including age, performance statistics, where they currently play and their potential. Player evaluation encompasses various methods, including game evaluation and statistical analysis. Both of these are integral components of the player assessment process. Despite thorough evaluations, the risk of misjudgement still remains, potentially resulting in financial losses for the team. Successfully acquiring the right players not only prevents financial losses but can enhances the player's market value and creates opportunities for future profitable sales. We will take a look at how to build a team from scratch rather than selecting individuals for an already existing team.

2 Description

Our project will take a look at building a starting line up that contains 11 players from the major 5 European leagues. The five leagues include the Premier League (England), Bundesliga (Germany), Ligue 1 (France), Serie A (Italy), and La Liga (Spain). These leagues are home to roughly 2500 players. When building a team, we use a specific layout called a formation, which dictates how players are positioned on the field. For our team, we'll use a formation that includes 1 goalkeeper, 4 defenders (who protect the goal), 3 midfielders (who help in both defense and attack), and 3 attackers (who focus on scoring goals). A diagram of the formation will be included in the Appendix. We aim to implement an age requirement to ensure our team remains competitive for many years ahead. This is motivated by the desire to have a balance of youthful and experienced players. A team with too many young players would lack experience while having many older players means the team may not be able to compete physically. With this balance you are able to future proof your team creating sustainable success over many seasons. Players market values will be required to make a sound estimate at how much we can expect to pay for a player. Considering all of the above we can explore and determine an optimal team given budget constraints.

3 Formulation

Our objective for the problem is to maximize a per player index score. We used the index developed by I.G Mchale and Phil Scarf(Scarf et al., 2012). This provides a way for us to measure how well a player performs. The index consists of different weights dependent on position. Stats needed for this Index are goals, assists, clean sheets, average minutes played per game and the average points per game the players team gets. With this objective function we hope to build the best team possible with respect to this player measurement system. Constraints involved will be age constraints, position constraints and team financial constraints. With financial constraints we are basing our team value off of the Premier League. We choose a budget of 350 million euros which puts us at 7th most expensive starting line up in the Premier League(Most Valuable Starting Line-up"2023). The output of our integer program is 11 players which will be our optimal team given the constraints.

Our formulation for this problem is as follow:

Variables:

Let *S* be the set of *n* players.

Let $S^{(p>1)}$ be the set of players that can only play more than one position.

Let P_i be the set of positions that player i can play. We define x_{ip} for $i \in S, p \in P_i$ to be a binary variable to indicate whether we choose player i to play position p. We can define another binary variable that indicates whether player $i \in S$ will play on the team, irrespective of the position.

Let
$$x_i = \sum_{p \in P_i} x_{ip}$$

Objective:

Our objective function is to maximize the sum of the index values, I_{player} , of players that will play on the team. Therefore the objective is max $\sum_{i \in S} I_i x_i$.

Constraints:

Team Composition:

The team formation that we decided to go with is having 3 forwards, 3 midfielders, 4 defence, and 1 goalkeeper. Let's denote these positions as *FW*, *MF*, *DF*, and *GK* respectively.

This leads to the following constraints:

Let \mathscr{P}_p be the set of players that can play position p for $p \in \{FW, MF, DF, GK\}$

Then:

$$\sum_{i \in \mathscr{P}_{FW}} x_{iFW} = 3. \sum_{i \in \mathscr{P}_{MF}} x_{iMF} = 3. \sum_{i \in \mathscr{P}_{DF}} x_{iDF} = 4. \sum_{i \in \mathscr{P}_{GK}} x_{iGK} = 1.$$

Age Constraints:

There are two kinds of age constraints we added. First, we want there to be a variety of ages of the team. For us, this means we want to choose at least three players aged at least 27 years old and at least three at most 24 years old. A second outcome we decided on was to have an average age of the players within 1 year of 25 years old. In other worlds, the average age of the players should be within the interval [24,26].

Let a_i denote the age of player i.

Let
$$a_{i,\geq 27} = \begin{cases} 1 & \text{if } a_i \geq 27 \\ 0 & \text{o.w.} \end{cases}$$
 and $a_{i,\leq 24} = \begin{cases} 1 & \text{if } a_i \leq 27 \\ 0 & \text{o.w.} \end{cases}$ for player *i*.

The constraints are therefore:

$$\sum_{i \in S} a_{i, \geq 27} x_i \geq 3, \sum_{i \in S} a_{i, \leq 24} x_i \geq 3, \text{ and}$$

$$\left| \frac{1}{11} \sum_{i \in S} a_i x_i - 25 \right| \leq 1 \text{ which we unwrap into:}$$

$$\frac{1}{11} \sum_{i \in S} a_i x_i - 25 \leq 1 \text{ and } \frac{1}{11} \sum_{i \in S} a_i x_i - 25 \geq -1$$

Position Constraints:

Each person can play one position. Therefore we add the constraint $\sum_{p \in P_i} x_{ip} \le 1$ for all $i \in S^{(p>1)}$.

Budget Constraints:

We want to also spend a certain budget *B* for the whole team. Our main value for *B* is 350 million euros, but we will also be calculating it for all possible values of *B*. We will use the market value of the players to calculate the total cost of the team.

Let m_i be the market value for player $i \in S$.

Then our constraint is $\sum_{i \in S} m_i x_i \leq B$.

Binary constraints:

 $0 \le x_{ip} \le 1$ and x_{ip} are integers.

Notes:

We will have x_i binary since $x_{ip} \ge 0$, $\sum_{p \in P_i} x_{ip} \le 1$ and $x_i = \sum_{p \in P_i} x_{ip}$. So $0 \le x_i \le 1$ and integer.

Summary:

The final IP is as follows:

$$\max \sum_{i \in S} I_{i}x_{i}$$

$$s.t. \ x_{i} = \sum_{p \in P_{i}} x_{ip}$$

$$\sum_{i \in \mathscr{P}_{FW}} x_{iFW} = 3$$

$$\sum_{i \in \mathscr{P}_{MF}} x_{iMF} = 3$$

$$\sum_{i \in \mathscr{P}_{DF}} x_{iDF} = 4$$

$$\sum_{i \in \mathscr{P}_{CK}} x_{iGK} = 1$$

$$\sum_{i \in S} a_{i, \geq 27}x_{i} \geq 3$$

$$\sum_{i \in S} a_{i, \leq 24}x_{i} \geq 3$$

$$\frac{1}{11} \sum_{i \in S} a_{i}x_{i} - 25 \leq 1$$

$$\frac{1}{11} \sum_{i \in S} a_{i}x_{i} - 25 \leq 1$$

$$\sum_{p \in P_{i}} x_{ip} \leq 1 \ \forall \ i \in S^{(p>1)}$$

$$\sum_{i \in S} m_{i}x_{i} \leq B$$

$$0 \leq x_{ip} \leq 1$$

$$x_{ip} \in \mathbb{Z}$$

4 Data

The primary dataset utilized for our project is sourced from Kaggle and encompasses statistics for 2,500 players from Europe's five major soccer leagues for the 2022/2023 season(Vinco 2023). It includes most of the comprehensive statistics necessary for the player index and the age constraints. It contains the general position of where a player plays (i.e defender, midfielder and attacker). However the dataset didn't have any goalkeeper statistics so we extracted this data using web scraping techniques(Player Advanced Goalkeeping"2023). We collected the player's market values from a dataset called Football Data from Transfermarktand joined them

together. Additionally, to ensure a focus on consistently performing players, we processed data to include only those who participated in at least 10 games and accumulated a minimum of 700 playing minutes during the season. This criteria reduced our dataset to approximately 1,000 players, helping eliminate outliers who had a high performance index but did not consistently participate throughout the season.

5 Player Index

As mentioned in section 3, in order to determine an optimal team, we must quantify players with an index score which will be used in the objective function. For our objective function, we strive to maximize the sum of our team's index scores, which would give us the empirically best team available subject to the given constraints. The index function we implemented is a modified version of the one detailed in a paper by Ian G. McHale, Philip A. Scarf, and David E. Folker. Here, they outlined a method of finding a performance index given a soccer player.

Our adaptation of their index calculations contains 5 sub-indices labeled I_1 , I_2 , I_3 , I_4 , I_5 . These are similar to the sub-indices defined in the paper with the exception of one. The paper defines a sub-index to quantify they performance of a certain player in the future given their previous statistics. They use a Poisson process to simulate the expected game events (eg. passes, blocks, crosses, red cards). We deemed that this would be computationally infeasible for our application and have decided to remove it. The papers index is also meant to give an index score for one specific game, however, since we wanted to get a more general depiction of a players performance, we have modified it to average all stats over a season. This gives us an average index score for a player over the whole 2022/2023 season. We define the weighted sum to be:

Definition 1
$$I_{player} = 100 \times (0.375I_1 + 0.125I_2 + 0.125I_3 + 0.0625I_4 + 0.0625I_5)$$

The specific weights attributed to each sub-index are detailed in the paper stated above and the method used to calculate them are outside the scope of this project. We continue on by defining each sub-index referenced above.

Definition 2
$$I_1 = \frac{\min_p}{90 \times 11} \times points_{avg}$$

Here, min_p is the average minutes per game played by the player, and $points_avg$ is the average points earned per game by the player's team (3 for win, 1 for tie, 0 for loss). This sub-index is meant to quantify a player's contribution to point scoring for their team as a ratio of minutes played. The denominator for this ratio is 90×11 which is the total number of minutes played by all players on the pitch. A player who plays many minutes on a team that consistently scores points would be favored by this sub-index.

Definition 3
$$I_2 = \frac{\min_p}{90 \times 11} \times 1.34$$

Here, min_p is defined in the same way as above and 1.34 is a constant derived in the referenced paper. This index is meant to quantify player contributions to a team outside of point scoring. The constant is meant to quantify the value a player brings to a team purely through the amount of minutes played in a game on average.

Definition 4
$$I_3 = goals \times 1.039$$

This sub-index quantifies a player's value based on the average number of goals they score in a season. The coefficient is calculated in the referenced paper as the average number of points one goal per game contributes to a team. Consistent scorers would be favorited by this index.

Definition 5 $I_4 = assists \times 1.039$

Similarly to I_3 this index assigns points to players who consistently assist goals. Since every assist results in a goal, the weighting of this stat is the same as goals.

Definition 6 $I_5 = clean\text{-}sheets \times weight_p$

weight_p varies based on the position the player plays. 0.585 for goalkeeper, 0.365 for defender, 0.15 for midfielder, and 0.071 for striker.

In soccer, a clean-sheet is defined as a game where your team does not concede a single goal. This is usually a testament to the defensive capabilities of a team, and as such $weight_p$ varies based on the role, giving the highest weight to a goalkeeper and lowest to a striker.

6 Analysis

For our Integer Program this is the optimal starting 11 that we get.

Robert Lewandowski, Marc Andre Ter Stegen, Danilo, Giovanni Di Lorenzo, Vincenzo Grifo, Mattia Zaccagni, Tasafusa Kubo, Jules Kounde, Ansu Fati, Folarin Balogun, Alejandro Balde output is shown in Appendix

6.1 Goalkeeper

Marc Andre Ter Stegen

With an index value of 104.49 and a market value of 35 million euros. We choose a goalkeeper who is very experienced and still has many years to come. In the 2022/2023 season he led the league in clean sheets with 26 and only conceded 18 goals in the league that season(Marc Andre Ter Stegen Stats"2023). With his total of 26 clean sheets he tied the La Liga record for most clean sheets in a season. We believe this is a great purchase as we are acquiring a great goalkeeper that will provide for our team for several seasons.

6.2 Defence

Danilo

With an index score 54.17 and a market value of 15 million Euros we have a relatively cheap defender at the age of 31. In the 2022/2023 he had a career year where he played 37 matches in the Serie A and had a total of 20 clean sheets. This is a very cost effective choice where he clearly has the experience as well as the ability to be in a well performing team.

Giovanni Di Lorenzo

Giovanni has an index value of 52.01 and a market value of 25 million euros, making him a reasonably priced defender. At 29 years old, he is in the peak of his athletic performance. He had 18 clean sheets in the 2022/2023 season where he was apart of the Serie A winning team Napoli where he played 37 matches(2022-23 Italian Serie a Standings, n.d.). League title winning experience is vital to a team looking to compete. With his success and years of experience at a high level this is a great addition.

Jules Kounde

Kounde has an index value of 57.19 and a market value of 60 million euros. While much more expensive than our previous defenders he is just 24 years old. Which leads us to believe he can be apart of our defence for many years to come and perform at a high level. He was apart of the Barcelona team that won the league in 2023 where he had 21 clean sheets and played 29 games (2022-23 Spanish LALIGA Standings, n.d.).

Alejandro Balde

Balde has an index value of 61.31 and a market value of 60 million euros. He is also apart of the La Liga winning Barcelona team where he played 33 matches and kept 23 clean sheets in those games(2022-23 Spanish LALIGA Standings, n.d.). He is just 19 years old which can lead us to believe he may not be experienced enough but, having that title winning experience can make a player mature from early on. With a player so young there is a variety of paths his career can take and even if he doesn't play with our team for his entire career it is likely that he will continue to improve his market value and potentially be sold at profit.

6.3 Midfield

Vincenzo Grifo

Grifo has an index value of 26.68 and market value of 12 million euros. He is a very budget friendly option as he had 20 goals and assists for SC Freiburg where he started 33 matches. SC Freiburg finished 5th which was there best finish in the Bundesliga within the last decade(2022-23 German Bundesliga Standings, n.d.). This Leads us to believe that Grifo played an important role within that team. At the age of 29 he is in the middle of his prime, which means he should continue to perform for the next few years.

Mattia Zaccagni

Zaccagni has an index value of 31.73 and a market value of 30 million euros. He had 17 goals and assists at the end of the 2022/2023 Serie A season. Where his team Lazio finished 2nd in the league(2022-23 Italian Serie a Standings, n.d.). At the age of 27 he is entering what looks to be the best years of his career and will be a good asset for the next few years to come.

Takefusa Kubo

Kubo has an index value of 22.76 and a market value of 25 million. He had 16 goal contributions while playing 35 games for Real Sociedad in La Liga. At the young age of 21 he clearly has what it takes to contribute to a team for the entirety of a season which is a great sign.

6.4 Forwards

Robert Lewandowski

Lewandowski has an index value of 30.43 and a market value of 30 million euros. He contributed 23 goals and 7 assists for the title winning Barcelona side(2022-23 Spanish LALIGA Standings, n.d.). He lead the entire league in goals scored that season which shows his worth as one of the best attackers in the world(LaLiga - List of Goalscorers 22/23, n.d.). He is 34 years old which leads us to believe he may only play at a high level for several seasons but he still provides great ability and experience for this team.

Ansu Fati

Fati has an index value of 19.39 and a market value 35 million euros. In the 2022/2023 season he contributed

11 goals and assists where he appeared in 36 games mostly as a substitute. He is 20 years old and still has a long way to go as a player it would be a great investment as he has tremendous upside. He was apart of the title winning Barcelona team.

Folarin Balogun

Balogun has an index value of 20.39 and a market value of 30 million euros. He had 21 goals and 3 assists while appearing 37 times for Reims. At the age of 21 he is another young player that is added to our team and he has great upside as scoring 20 goals in a season is a great achievement. With a relatively cheap market value this is a solid buy and allows our team to invest in our future.

6.5 Overall Team

Our solution accomplished what we wanted given our formulation. For our under 24 players, we had a requirement that there was at least three in the team. The players under 24 in the solution were what we were looking for: players who have a high upside and are currently performing. The importance of them performing currently is that we want to compete as soon as possible, which requires players who are ready to play. As for our over 27 requirement, it brought the expertise we were looking for. From leaders in clean sheets and goals we have the excellence required to compete for trophies. Overall this solution encompasses our idea of a great team given a budget.

6.6 Going forward with this solution

In our solution we found that we had selected many players who were clearly talented and contributing to successful teams. We also had a great balance of age with younger players who will hopefully be apart of our team for years to come and older players who bring experience and expertise. With this balance we hope to build a team that can compete currently and in the future. But we must realize that this isn't the only thing it takes to make a great team competitive. It also comes down to many factors such as coaching, training and how well players truly adjust to moving teams. This was a great start at selecting a team, it is important to note that building a team isn't as simple as selecting players and letting them play for themselves. It takes great commitment from the entire organization to build a team that wins titles.

7 Simulation

We decided to test out the team on EAFC 24 where we could use the team in manager career mode. To make this simulation fair and mostly dependent on our selected team we chose Luton town as a base, which is the least valuable premier league team. All that we were in control of was setting the starting lineups. It is important to note that if a player from our optimal team was injured they would be replaced by a substitute with the substituting player being inferior to whomever they replaced. We simulated the season after the data we had covered and these were the results. The league table after simulation is found here.

We had a very solid season landing 5th out of 20 in the Premier League. Which is great considering our team budget for our starting 11 of 350 million euros would have landed us 7th. In terms of starting 11 value we can now take a look at how our players performed individually found here.

The player that contributed the most goals and assists this season was Kubo. He played every game of the season and was a very valuable member of our team. Ter Stegen and the defence had a great season as well where he

had 13 clean sheets which is great for his debut season in a new league. A player who didn't perform as expected was Robert Lewandowski he had 11 goals and 5 assists which was a step back from his previous year. But this regression in stats was expected as he is the oldest player in the team but he still provided for the team despite the fall off. Our team had some success in its first season and it was a great sign of what this team can do.

8 Cutting Planes

8.1 Approach

As an extension, we analyze how different budget values affect team selection, we used cutting planes to determine the optimal team for each budget range. We started by not including a budget constraint, and found that our optimal team's total cost was 710 million euros. Since this optimal team is feasible and none of our other constraints rely on the budget value, it will remain an optimal solution for all budgets greater than or equal to 710 million euros. So, for a budget \in [710e+6, ∞), our initial IP finds an optimal team. Then, we add a cutting plane, limiting the total money spent on players to 710 million euros minus 1000. We subtract 1000 because Gurobi does not support strict inequality constraints, and all player market values are multiples of 1000, so this does not eliminate any solutions. Similarly, the new optimal solution will be optimal for any budget in the range [optimal solution cost, 710e6). Then, the next cutting plane constrains the budget to the total cost of the optimal solution minus 1000, and so on. This process continues until our integer program is infeasible because the budget is too low to find a team that satisfies the full set of constraints. This method yielded 347 possible budget ranges.

Moreover, we attempted subtracting smaller numbers such as 100 euros. This led to greater floating point imprecision errors where either the solution got stuck or we would not create a full team of 11

In addition, for efficiency, instead of adding a new budget constraint at each iteration, we deleted the previous budget constraint and replaced it with the new budget. This is equivalent since a lower budget constraint will satisfy any existing higher budget constraint. This reduced our budget analysis runtime from 1 minute to 20 seconds. The full code is under gurobi_implementation.ipynb

8.2 Pseudocode

```
Starting IP (P): full formulation with no budget constraints
x: player indicator variable vector
While (P) is feasible:
    x' = optimal solution for (P)
    Total cost = (market values of players)*x
    new_budget = total cost - 1000
    Delete current budget constraint to (P)
```

8.3 Cutting Planes Discussion

As expected, the total index value of an optimal team is non-decreasing as budget increases. A graph of total index value against team cost can be seen in the graph here

add new budget constraint to (P): \sum (market values of players)* $x \le$ new_budget

Noticeably, the total index value of a team increases much more sharply when making increases to a lower budget rather than increasing an already substantial budget. This is logical, since the distribution of player market value is approximately exponential. Graph

Thus, after a certain minimum budget, additional funds make little difference to the players one can afford.

Interestingly, we can also infer which budget results in the 'best value team', i.e. the team with the highest total index score to total cost ratio. Since index value sees a greater increase at the lower end of budgets, we found that the best value team cost 13.2 million euros, which is much lower than the average starting 11 cost of 350 million euros in the premier league. Graph

The best value team we can find is listed in the Appendix. We found with simulation that this team wasn't up to par. In the simulation the team was bottom of the league and it resulted in the manager of the team getting fired. Which it is important to realize that you can't cut costs on your team too much or else it will severely affect performance.

9 Next Steps

9.1 Positions

The first major thing that would improve the accuracy of these solutions is using a dataset with exact positions. Currently, our dataset only categorizes players broadly as defenders, midfielders, or attackers, without distinguishing between more specific roles such as strikers or wingers within the forward positions for example. As a result our output may have players out of position which will likely affect their performance. With exact positions we can make our integer program have more constraints which would decrease our feasible region but give us a much better performing team.

Additionally, our dataset includes classifications for players' secondary general positions. For example, a player who primarily serves as an attacker but has also played as a midfielder in some games would be listed under both roles. This nuance is vital because our scoring index heavily favors metrics such as goals and assists, which are more frequently achieved by attackers. As a result, a player who mostly plays in attacking positions but is also listed as a midfielder might be selected as a midfielder in our model due to their high scoring potential. This selection is noteworthy because midfielders are typically evaluated by a broader range of contributions, such as passing accuracy, interceptions, and their ability to control the flow of the game. This disparity in role expectations underscores the importance of incorporating precise positional data to truly optimize team configuration and performance.

9.2 Not all leagues are equal

It is important to note that when looking at a players stats we need to consider what league they play in to be a factor. Different leagues may vary significantly in the balance and strength of competition, affecting the reliability of a player's stats. One way we believe this can be incorporated is looking to apply some weight to a players stats depending on the league they play in, adjusting their stats to reflect the competitive context more accurately. This is similar to the position-based weights for the index. Even if a player is performing exceptionally well in a less competitive league, this multiplier ensures their achievements are appropriately valued. Implementing this weighting system will enhance the accuracy of our player evaluations and the selection process in our integer program.

9.3 More leagues

When building teams they typically have a scouting report for players all across the world. We are currently limited to just players in Europes big 5 leagues. We would be missing any players in the lower divisions and ones that haven't made it out of their respective continent yet. This is how teams find future stars at a much lower price as they haven't been discovered yet. So, as a team we want to also find these future stars at cheaper prices. It allows us to do much more with our budget.

9.4 Chemistry of team

We currently cannot measure how well the team will work together. However it may be possible to gather new data that could address this issue. We could take a look the number of passes each player gives and receives, as well as where they pass the ball. This could affect how players play together in game.

For example, consider a scenario where we have a defender that frequently passes the ball to other defenders. If the other defenders aren't good at receiving the ball and typically receive fewer passes. That would be important interaction to observe and see how well each player may work together.

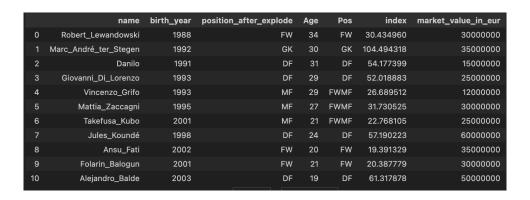
10 Appendix

10.1 Team Formation



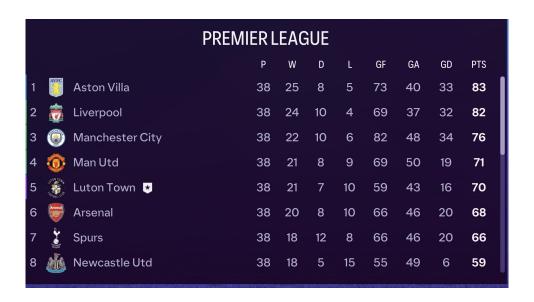
Figur 1: Formation used for our team

10.2 Team Output

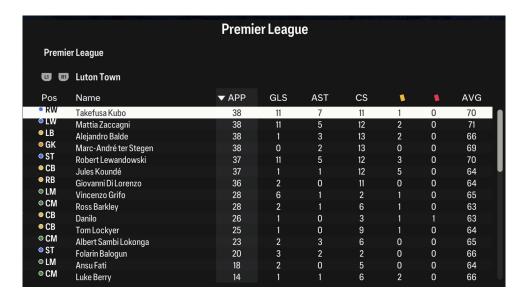


Figur 2: Output from IP

10.3 Simulation Pictures

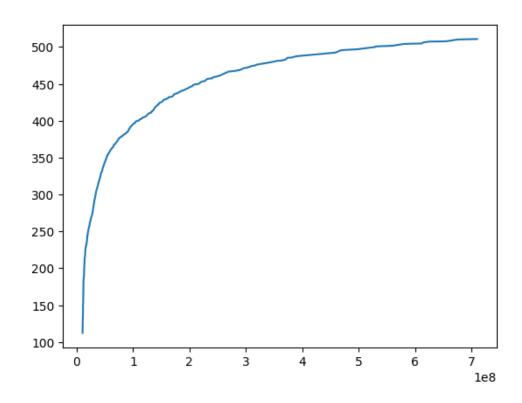


Figur 3: Position in the league

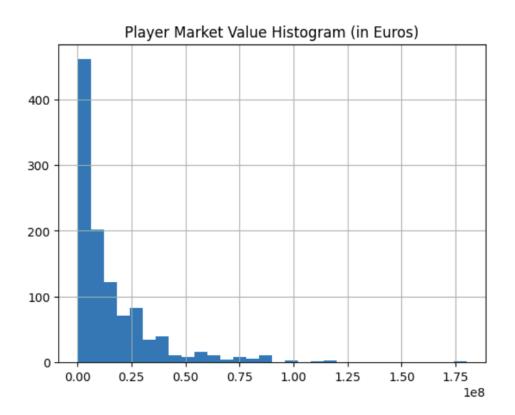


Figur 4: Player stats

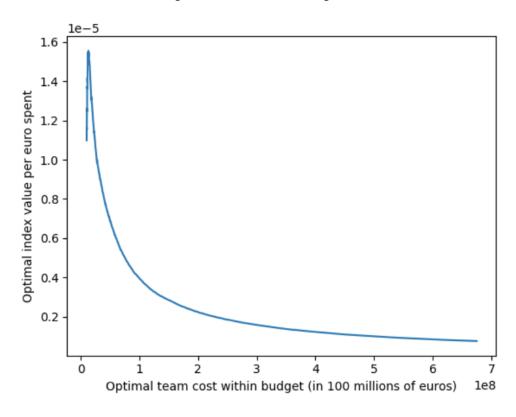
10.4 Cutting Planes



Figur 5: Optimal index of a team vs optimal budget



Figur 6: Market Value histogram



Figur 7: Optimal index per Euro vs optimal team cost by budget



Figur 8: Best Value team

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